

## **Long Memory Properties and Agents' Behavior: An Interesting Connection**

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### **Abstract**

The aim of this work is to establish an interesting connection between the behavior of economic agents and the long memory features that generally occur in a wide set of time series found in economic/financial problems. It is shown that heterogeneity between agents, local interactions and spatial complexity plays a major role in the rise of long memory features, by means of extensive usage of computational multi-agent based models, stochastic analysis and Monte Carlo simulations. Despite the fact that heterogeneity is a widely known characteristic that affects the rise of long memory, the other two factors are not.

**Keywords:** long memory; agent based models; complexity; heterogeneity

### **Introduction**

Long Memory Processes are stochastic processes that exhibit non-exponential decay in their respective autocorrelation functions, as usually observed in what can be called “short memory processes”. Therefore, the main feature of this type of stochastic process is a hyperbolic decay in its respective autocorrelation function, which points out that perturbations occurred far away in time are still able to explain part of the current state of the system.

Consequently, this kind of stochastic process exhibits persistence that is neither consistent with the presence of a unit root nor with its complete absence. Hence, in order to have the necessary flexibility to deal with this apparent dilemma, it is introduced a fractional difference coefficient, which

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tries to accommodate stochastic processes between those with a unit root process ( $d = 1$ ) and those with no unit root ( $d = 0$ ).

Thus, a long memory stochastic process can be defined as:

$$x_t \cdot (1 - L)^d = \epsilon_t$$

where  $\epsilon_t$  is a stochastic disturbance (white noise),  $L$  the lag operator,  $x_t$  the contemporaneous observation of the stochastic process and  $d$  the fractional difference operator. Furthermore,  $(1 - L)^d$  can be defined as:

$$(1 - L)^d = \sum_{k=0}^{\infty} \binom{d}{k} \cdot (-L)^k = 1 - dL - \frac{1}{2} \cdot d \cdot (1 - d) \cdot L^2 - \frac{1}{6} \cdot d \cdot (1 - d) \cdot (2 - d) \cdot L^3 - \dots$$

The study of this kind of phenomenon is a relatively old field in Mathematics and Physics, which started to be investigated right at the beginning of the 1950s, by Hurst (1951), followed by Mandelbrot & Wallis (1969) and McLeod and Hipel (1978), among others. Then, during the 1980s and 1990s the field grew almost exponentially, where the most important works were those written by Hosking (1981), Geweke & Porter-Hudak (1983), Robinson (1994) and Baillie (1996). The main focus of the pioneer works was the search for empirical evidences over a different range of problems, while the focus of the posterior works was the mathematical/statistical analysis of such properties in terms of its stochastic components and the development of proper mathematical tools and statistical tests aiming at the calculation of the fractional difference coefficient.

Given that, most of the papers published since then focused on empirical evidences of long range dependency in a wide set of different fields, from Physics to Finance, on propositions that enhance the computational calculations of models which exhibit these stochastic properties and on the development of the theoretical mathematical toolset used to build and analyze such models.

However, to the best of the present authors' knowledge, since then there are no other papers that discuss the aspects of such stochastic properties origins by establishing such link with Dynamic Complex Systems, agents' individual behavior, aggregate behavior and other related fields, integrated with the possibility of doing computational simulations that are able to generate such behavior.

Hence, the main idea of this paper is to discuss possible origins of such phenomenon by running computational simulations of the interactions between single individuals (called agents), which produce local and global interactions that are studied in terms of its stochastic properties. Moreover, this paper aims to show and discuss that long memory properties are not necessarily resultant from long memory

behavior of individual agents nor from social/economic frictions. Furthermore, it extends the knowledge of agent behavior complexity (initially restricted to heterogeneity), by adding two other important factors: spatial complexity and topology (i.e. how the agents are interconnected).

The importance of the present work is due to the fact that the correct specification of stochastic processes is very important. Its specification affects the description of the autocorrelation structure of a wide range of problems, such as asset pricing, macroeconomic modeling and other time series phenomena. The misspecification of such features may induce very different results in long term, affecting the way that optimal policymaking may be conducted, since these effects last longer than short memory.

To accomplish this goal, in such computational models, the agents must only explicitly have short memory relationships with their respective past states. Thus, it should be possible to show that long memory properties arise not because the agents may have a memory unit which guides them in their respective actions (behavior), as one may think in terms of traders pricing an asset according to the present and their perception of a fair price based on their long experience; but as a result of the aggregate behavior of them, as a consequence of the complexity emergence, pointing back to the seminal works of Mandelbrot & Wallis (1969) and establishing an interesting link with the growing field of Complexity, as in Wolfram (2002), Monteiro (2011, 2014), among others.

Consequently, the behavior of agents in such systems would be somewhat affected by disturbances occurred in a far past, but not explicitly derived of individual long memory behavior, which affects directly the development of optimal control policies for such kind of systems.

Keeping that in mind, in this work, three different computational models are presented and simulated, showing that long range dependency may simply arise from the interactions between the agents, establishing what can be called “long memory emergence”.

On the other hand, none of these models were developed for this work. Their respective authors separately made them for specific purposes and that is why the present authors have decided for such strategy (of picking models made by third parties). Instead of building models (which usually takes a considerable amount of time to make them work properly) that might contain biases in terms of finding such long memory properties – as a consequence of the present paper idea – they were chosen, simulated (in their respective platforms) and analyzed using the R Statistical Package.

In the following section it is presented a brief discussion about the capability of the current fractional difference coefficient estimators to distinguish apparent long memory processes from simple autoregressive processes – due to the fact that autoregressive processes can be seen as a truncated version of long memory processes. Then, in the other three next sections, each one of these three models are presented, simulated and discussed in terms of the stochastic properties found over the results obtained, while pointing out possible reasons for such results as a consequence of agents' heterogeneity, local interactions and spatial complexity. After that, is presented a final section containing a brief conclusion of evidences towards the emergence of long range dependency as a result of other kind of interactions beyond explicit long memory behavior on individuals.

### Methodology

In order to test the possibility of having a long range dependency over time, it is proposed along this paper the usage of two statistical tests: Local Whittle Estimator as in Robinson (1995) and Geweke & Porter-Hudak (1983) – from now on GPH. Thus, in order to discuss briefly how both estimators work, it is necessary to define what is an invertible and stationary process.

As can be seen in Kumar (2014), the GPH estimator is based on the slope of the spectral density function of the fractionally integrated time series around  $\lambda \rightarrow 0$  through a simple linear regression based on the periodogram. The periodogram is defined as:

$$I(\lambda_k) = \frac{1}{2\pi T} \left| \sum_{j=1}^N X_j e^{ij\lambda_k} \right|^2$$

where  $I(\lambda_k)$  is the  $k^{\text{th}}$  periodogram point and it can be defined as the squared absolute values of the Fourier Transform of the series.

Having calculated the periodogram, the final step is to estimate the fractional difference coefficient  $d$  by estimating the following regression using Ordinary Least Squares:

$$\log(I(\lambda_k)) = a - d \cdot \log\left(4 \cdot \sin^2\left(\frac{\lambda_k}{2}\right)\right) + \varepsilon_k$$

where  $\lambda_k = \frac{2\pi k}{N}$  is a constant and  $\varepsilon_k \sim N(0, \sigma_k)$ .

On the other hand, the Local Whittle Estimator depends also on the periodogram, as defined right above, but consists of a semiparametric estimate, which is carried out by the minimization of an objective function, as in Künsch (1987):

$$R(H) = \log\left(\frac{1}{m} \cdot \sum_{k=1}^m \{\lambda_k^{(2H-1)} \cdot I(\lambda_k)\}\right) - \frac{(2H-1)}{m} \cdot \sum_{k=1}^m \log(\lambda_k)$$

where:

- $H$  is any admissible value
- $m = \begin{cases} \frac{1}{2}N & \text{if } n \text{ is even} \\ \frac{1}{2}(N-1) & \text{if } n \text{ is odd} \end{cases}$
- $d = H - \frac{1}{2}$ , being  $H$  the Hurst exponent.

Keeping that in mind, both estimators are applied over generated time series, which are result of the simulation of each model explained. Hence, for  $0 < d < 1$ , the time series display long memory properties.

Nonetheless, it is also important to notice that despite the fact that ARIMA processes can be seen as truncated fractionally integrated processes, the results of both tests do not suggest fractional difference coefficients where they do not exist.

About the first part of the affirmative above, if a fractionally integrated process can be written as:

$$\begin{aligned} x_t \cdot (1-L)^d &= \sum_{k=0}^{\infty} \binom{d}{k} \cdot (-L)^k \cdot x_t \\ &= \left(1 - dL - \frac{1}{2} \cdot d \cdot (1-d) \cdot L^2 - \frac{1}{6} \cdot d \cdot (1-d) \cdot (2-d) \cdot L^3 - \dots\right) \cdot x_t \end{aligned}$$

obviously, simple autoregressive processes can be seen as their respective truncations of order  $p$ .

Moreover, in order to obtain an interesting proof in terms of the statistical tests performance without any further mathematical abstraction, a Monte Carlo based test was carried out, with 100 simulations for each time series specification with 2000 observations.

Only first order processes were tested, since none of the 2<sup>nd</sup> order autoregressive models suggested any spurious fractional difference coefficient. The same occurred for 3<sup>rd</sup> order processes and so on.

So, basically, it was simulated AR(1) processes with the first order term ranging from  $\phi_1 = 0.9$  to  $\phi_1 = -0.9$ , and the results are shown in Figure 1, in terms of the suggested fractional difference coefficient.

Also, it is important to keep in mind that an AR(1) process (without constant term) can be represented as a model of the form:

$$x_t = \phi_1 \cdot x_{t-1} + \varepsilon_t$$

If  $\phi_1 = 1$ , the model becomes a pure random-walk process (an ARIMA(0,1,0) model).

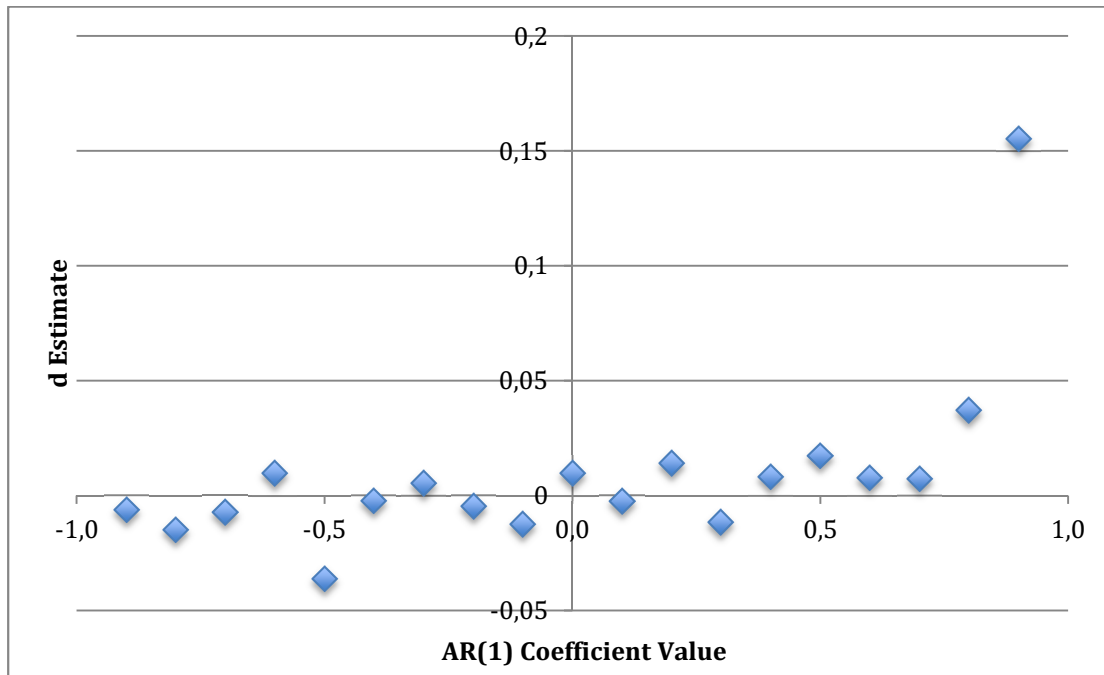


Figure 1: SPURIOUS ESTIMATES OF  $D$  VERSUS TRUE AR(1) COEFFICIENT VALUES

As can be seen, only high positive values of the autoregressive coefficient can produce potential spurious results in terms of the estimate of fractional difference coefficients. But still, these spurious effects are not significantly large. The main explanation for such occurrence is the fact that as  $\phi_1 \rightarrow 1$ , the model becomes more similar to a random-walk process. Hence, in terms of inference of  $d$ , it should approximate to 1.

Furthermore, it is also important to mention that both statistical tests produced the same results for sufficient long series, in this case, 2000 observations, and the results did not change if a constant was included.

In order to complement this first analysis, it was simulated a case where  $\phi_1 = 0.95$ , resulting in  $d = 0.3428$ . Hence, it is possible to conclude that there is an exponential decay behavior in terms of a spurious fractional difference parameter estimate versus the AR(1) coefficient value.

Hence, only first order autoregressive processes with an AR coefficient close to 1 (larger than 0.9) can lead to possible distortions in the inference of the fractional difference coefficient, which is something that must be taken into account when evaluating data generating processes.

## A Multi-Agent Prediction Market based on Boolean Networks

The main idea of this first model is to simulate the dynamics behind the interactions between individuals, an external source of information and a market clearer, which aggregates the individual beliefs. It was taken from Jumadinova, Matache & Dasgupta (2011).

As the authors point out:

*“Prediction markets have been shown to be a useful tool in forecasting the outcome of future events by aggregating public opinion about the events’ outcome. Previous research on prediction markets has mostly analyzed the prediction markets by building complex analytical models. In this paper, we posit that simpler yet powerful Boolean rules can be used to adequately describe the operations of a prediction market”.*

The basic structure of this model is composed of individual agents which update their respective beliefs according to a Boolean based rule, where they assume a binary belief state: 1 when they believe a specific event will happen; 0 when they believe that a specific event will not happen. The factors that are weighted in order to assume one or another state are:

- The individual past belief state, given by  $S(t - 1)$
- Overall average of the individuals past belief state (condensed into “prices” between 0 and 1 – continuous variable), given by  $P(t - 1)$
- External Information, represented by a Bernoulli Random Variable, given by  $B(t)$ , with a probability  $q$  obtaining a 1 and  $(1 - q)$  of obtaining a 0

The overall average of individuals beliefs are condensed into prices according to the following mathematical expression:

$$P(t) = \sum_{i=1}^N \frac{S_i(t)}{N}$$

Furthermore, there is a mathematical function that updates the belief state according to the following expression:

$$S(t + 1) = \begin{cases} w_1 \cdot S(t) + w_2 \cdot B(t) + w_3 \cdot P(t) > z \rightarrow 1 \\ w_1 \cdot S(t) + w_2 \cdot B(t) + w_3 \cdot P(t) \leq z \rightarrow 0 \end{cases}$$

where:

$$\sum_{i=1}^3 w_i = 1$$

and:

$$w_i \in [0,1]$$

Moreover,  $z$  is the individual bias, generated randomly for each agent.

In the implementation of this code, for the purpose of this paper, in order to generate heterogeneity between the individuals, it was imposed that  $z \sim N(0.5, 0.1)$ , fixed at the first simulation step.

For simplicity, in this paper it was adopted a probability  $q = 0.5$ ,  $w_1, w_2 = 0.3$  and  $w_3 = 0.4$ , in order to avoid any apparent bias in the generated time series and any very strong autocorrelation over individual past states.

Thus, basically, this set of rules represents a simple Boolean Network, where all agents are interconnected (which simulates a situation of synchronous information and perfect information symmetry), simplified by the existence of an external “Market Maker” agent, which condensates all agents (nodes) beliefs into something that resembles a price. On the other hand, the state of each agent does not depend on any spatial position, since they are all virtually connected and information flows instantaneously, resembling individuals interconnected by a mechanism such as Internet.

For the purpose of this paper, this model was implemented using the software Insight Maker and 25 simulations were carried out, where each one generated price time series that encompassed 1000 ticks. An example of a resulting series is shown in Figure 2.

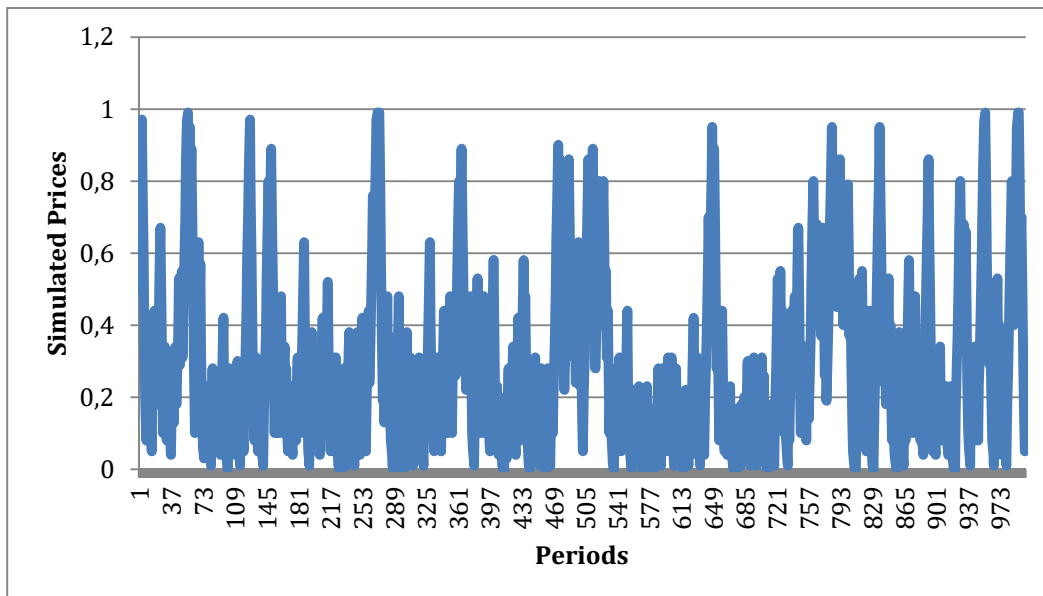


Figure 2: SIMULATED PRICES OVER TIME



Thus, it was calculated the Künsch (1987) and GPH (Geweke; Porter-Hudak, 1983) estimates of fractional difference coefficients over these price series, in order to test the presence of long memory components.

The average Künsch (1987) estimate for the fractional difference coefficient for the 25 simulations was 0.4899566, while the average GPH (1983) estimate was 0.2357935. If taken into account the fact that the past state is Boolean and the autoregressive part of the function is still weak (lesser than 0.9), both results provide strong evidences towards the presence of long memory components in this kind of process. The distribution of the fractional difference estimates (GPH) is described in Figure 3.

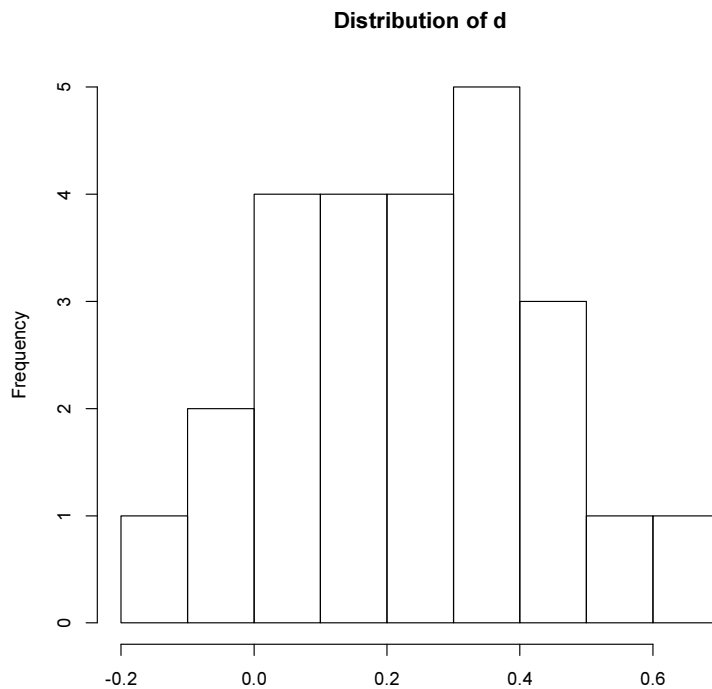


Figure 3: HISTOGRAM OF THE PARAMETER  $D$

According to Figure 3, it is clear that this process exhibits long-memory properties, avoiding any spurious result from a single simulation, as it relies on a Bernoulli random variable to generate part of the stochastic fluctuations.

In Figure 4 it is shown the distribution of the fractional difference estimates, according to Künsch (1987).

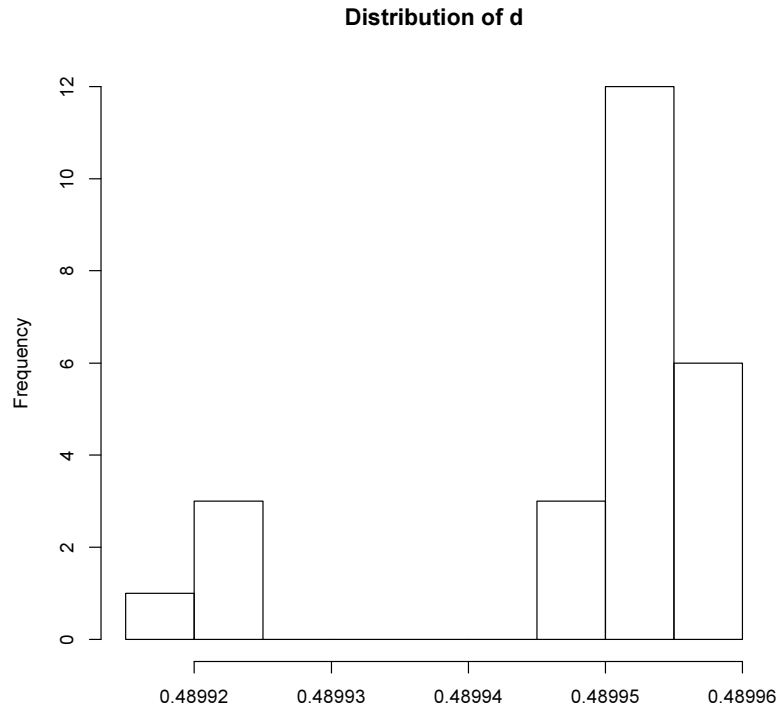


Figure 4: HISTOGRAM OF THE PARAMETER  $D$

According to Figure 4, it is important to notice that the shape of the distribution is completely different from the GPH (1983) estimates. Still, according to these results, it suggests the presence of long range dependency.

Nonetheless, in order to reduce the heterogeneity between the simulated individuals, this experiment was repeated using  $z \sim N(0.5, 0.05)$ , fixed at the first simulation step.

When the heterogeneity is reduced, the system behaves completely different. It rapidly converges towards a low-level price or a high-level price, resembling a white noise around these levels, as shown in Figure 5.

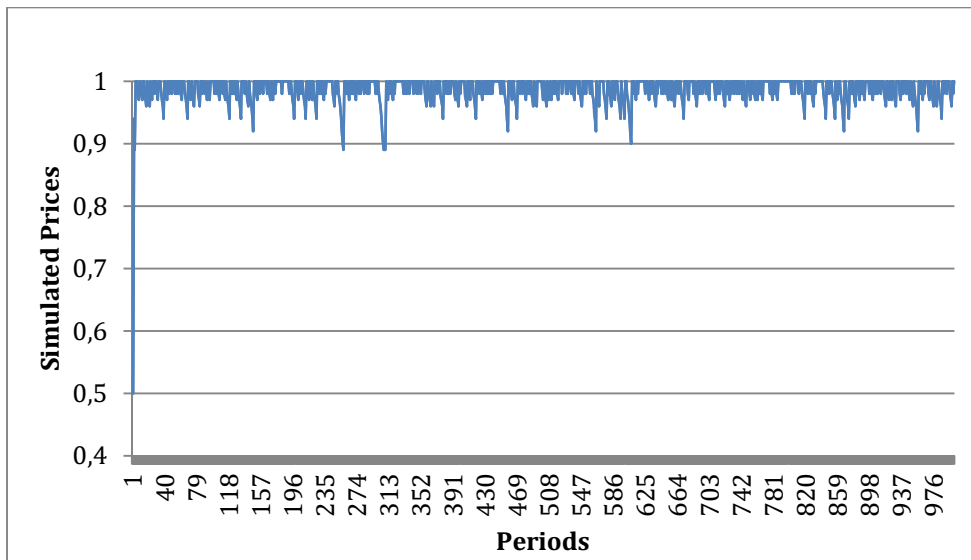


Figure 5: SIMULATED PRICES OVER TIME

With a probability  $q = 0.5$  of having a positive or negative external information, according to the mentioned Bernoulli variable, the system converges rapidly towards a high-level or a low-level price. In the case of Figure 5, it converged towards a high-level price.

Having estimated the fractional difference coefficients, it was obtained an average Künsch (1987) estimate of 0.3633277, while the average GPH (Geweke; Porter-Hudak, 1983) estimate was 0.02466105.

Again, if taken into account the fact that the past state is Boolean and the autoregressive part of the function is still weak (less than 0.9), the GPH results provide weak evidences towards the presence of long memory components in this kind of process – in this case, it suggests a White Noise Process. On the other hand, the Künsch (1987) estimates provide evidences towards the presence of these long range dependencies.

The distribution of the fractional difference estimates (GPH) is described in the Figure 6.

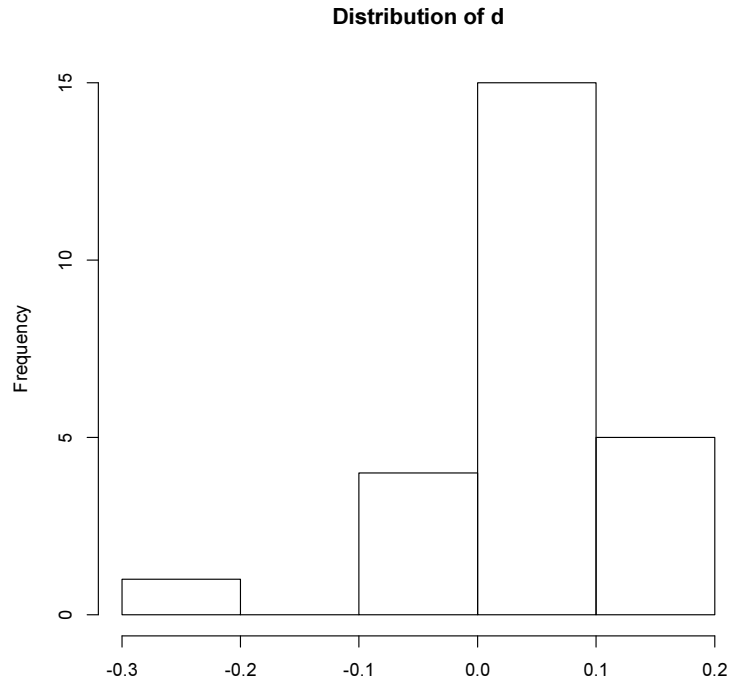


Figure 6: HISTOGRAM OF THE PARAMETER  $D$

In contrast to the previous distribution, in Figure 6 is clear that this process does not exhibit long memory properties. In Figure 7 it is shown the distribution of the fractional difference coefficient estimates according to Künsch (1987).

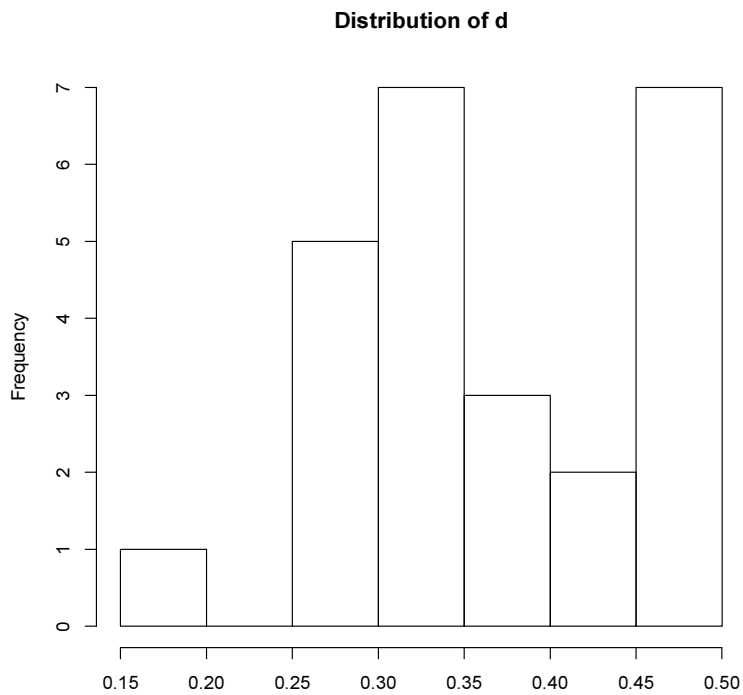


Figure 7: HISTOGRAM OF THE PARAMETER  $D$

When analyzing Figure 7 it is important to notice that its shape is completely different from those obtained in the previous cases. This distribution suggests the presence of a long range dependency in the analyzed stochastic process.

Another interesting fact is that the shape of the simulated price distribution is completely different in both cases, as seen in Figure 8.

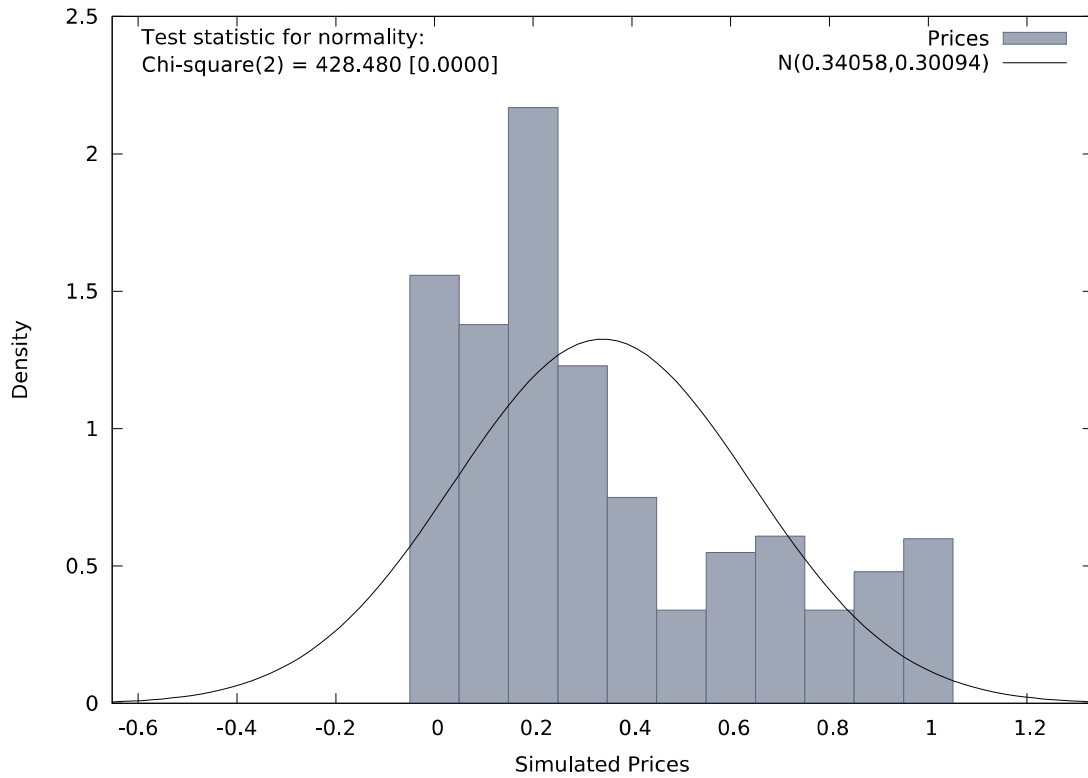


Figure 8: HISTOGRAM OF SIMULATED PRICES

In Figure 8, that synthesizes the distribution of the prices in the first case, it can be seen that the prices are very far from a normal distribution, with considerably high *fat tails*, asymmetry and so on, resembling a Power Law-like distribution.

Except for the percentile between 0.2 and 0.3, which has a very strong peak, there is a typical exponential decay, which is one of the most important features in a Power Law distribution.

Furthermore, the characterization of such behavior is very important, due to the fact that this is one of the most notorious aspects present in self-similar processes, which are naturally one of the emergent properties of complex systems.

Thus, as it is widely known that Power laws are an important characteristic of self-similar processes, and on the other hand, knowing that long-range dependency can arise from such processes,

this is one more interesting finding towards the obtainment of empirical evidences of the presence of such property.

This same procedure was applied to other set of experiments made in the complementary case, as it follows in Figure 9.

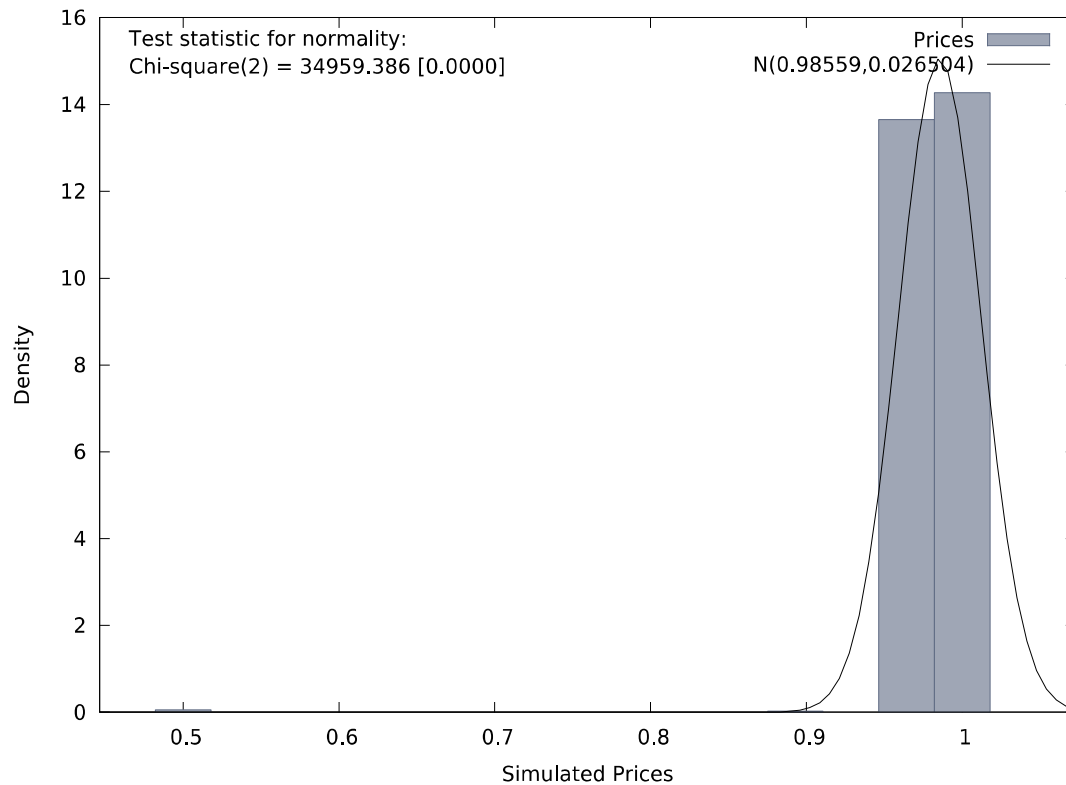


Figure 9: HISTOGRAM OF SIMULATED PRICES

In Figure 9, which synthesizes the distribution of the prices in the second case, it can be seen that the prices are not that far away from a normal distribution, despite the fact that the statistical test for normality rejects a normal distribution.

This distribution is far more symmetric than the previously presented and does not exhibit huge fat tails, not suggesting a Power Law like distribution, which is an interesting evidence in terms of the absence of long range dependency.

Hence, what can be said in terms of agent-based models is that the introduction of heterogeneity among the agents is an important factor in terms of the existence of long memory properties in any process. When heterogeneity is considerably reduced, it seems these properties vanish, at least in this model.

### **Rule 110 – A Turing Complete Elementary Cellular Automaton**

This is one of the most intriguing and beautiful pieces of computer software ever written. With a set of simple 8 bitwise rules, one is able to, in principle, compute any calculation or emulate any computer program, as proved by Cook (2004) and conjectured by Wolfram (1985).

Furthermore, with this set of simple rules, one is able to generate very complex patterns that seem to evolve chaotically, depending on the initial conditions of the system. According to Wolfram (2002), this kind of system exhibits a behavior that is neither completely stable nor completely chaotic, generating localized structures that appear and interact in various complicated-looking ways.

In order to explain this specific algorithm, it is worth explaining what is a Cellular Automaton first. According to Wolfram (2002):

*“A cellular automaton is a collection of "colored" cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells. The rules are then applied iteratively for as many time steps as desired.”*

Nonetheless, according to the same author, it is also important to notice that:

*“Cellular automata come in a variety of shapes and varieties. One of the most fundamental properties of a cellular automaton is the type of grid on which it is computed. The simplest such "grid" is a one-dimensional line. In two dimensions, square, triangular, and hexagonal grids may be considered.*

[...]

*The number of colors (or distinct states)  $k$  a cellular automaton may assume must also be specified. This number is typically an integer, with  $k=2$  (binary) being the simplest choice. For a binary automaton, color 0 is commonly called "white" and color 1 is commonly called "black". However, cellular automata having a continuous range of possible values may also be considered.”*

Keeping that in mind, the main idea of this second experiment is to show that there is another interesting source of complexity in terms of the rise of long memory processes.

If in the previous model all agents behave heterogeneously and globally – since their respective state did not depend on any specific neighborhood range, but relied on all agents' states (global

interaction) –, in cellular automata it is set the opposite situation. All agents behave locally and homogenously, where each cell represents an agent.

Thus, knowing that Rule 110 Cellular Automaton is wide known to exhibit complex behavior, in terms of the evolution of its respective states, it was tested if such elementary cellular automaton was also capable of generating time series with long range dependency.

Using the following set of logical rules shown in Figure 10, this discrete dynamic system was simulated for 2000 iterations, over 25 different random initial conditions, with a Moore neighborhood of 200 cells (where borders are interconnected as a continuum of cells).

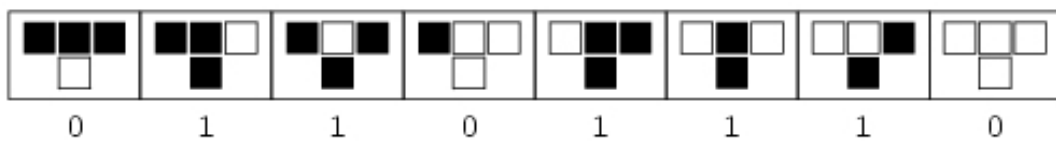


Figure 10: EVOLUTION RULE

Consequently, in order to analyze long range dependency, it was considered the amount of black cells, which are represented by ones, in each time step (iteration) of the dynamic system, as it follows from the expression below:

$$S(t) = \sum_{i=1}^{200} C(i, t)$$

where  $C(i, t)$  represents the value of the cell at position  $i$  at time step  $t$ . Hence,  $S(t)$  can be seen as a time series that has apparent stochastic behavior, as shown in Figure 11.

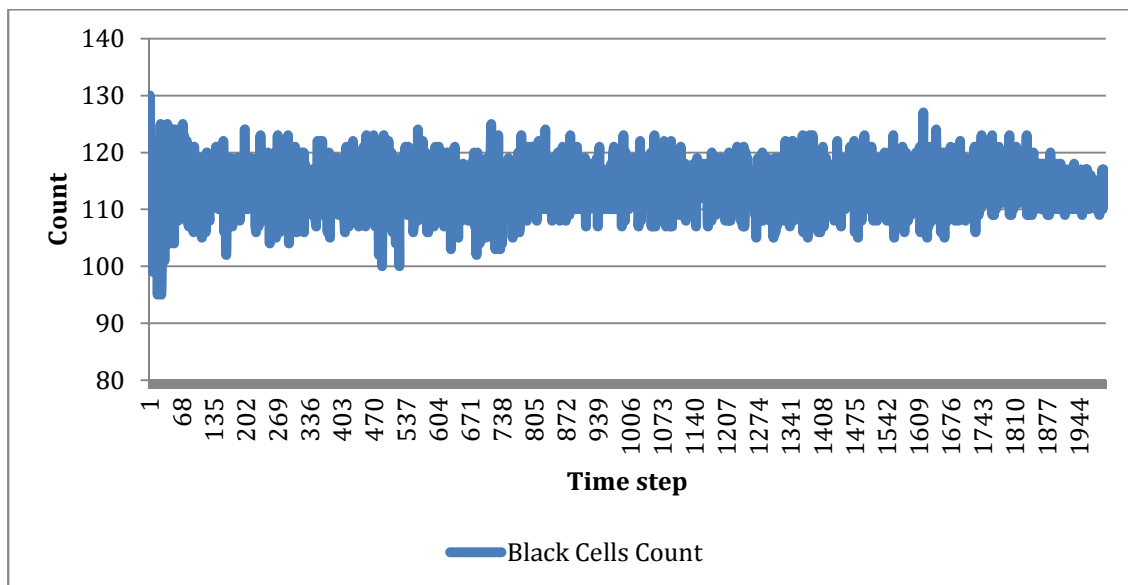


Figure 11: GENERATED TIME SERIES OF BLACK CELLS COUNT



In order to study the stochastic properties of this system, the same procedures considered in the previous experiment are performed in this one. First, in Figure 12 it is shown the distribution of the fractional difference parameter estimate, according to GPH (Geweke; Porter-Hudak, 1983), for each simulation made, which is a consequence of a specific random initial condition.

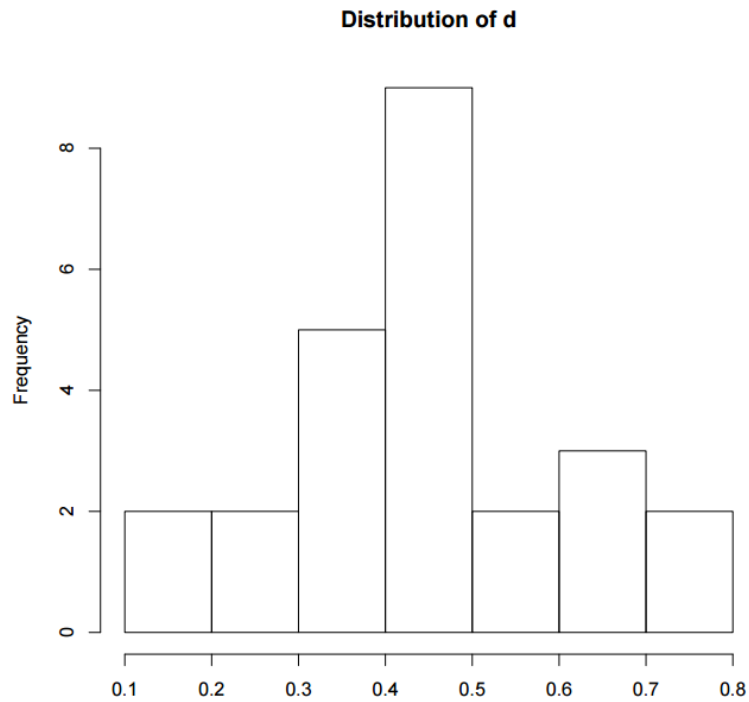


Figure 12: HISTOGRAM OF THE PARAMETER  $D$

As can be seen in the distribution of the estimated  $d$  parameters in Figure 12, there is a strong evidence in favor of long range dependency. The mean of this distribution is 0.4459731 and its standard deviation is 0.1614409, being statistically different from zero.

Furthermore, the inference of the  $d$  parameter was also made using the Künsch (1987) procedure, as seen in Figure 13.

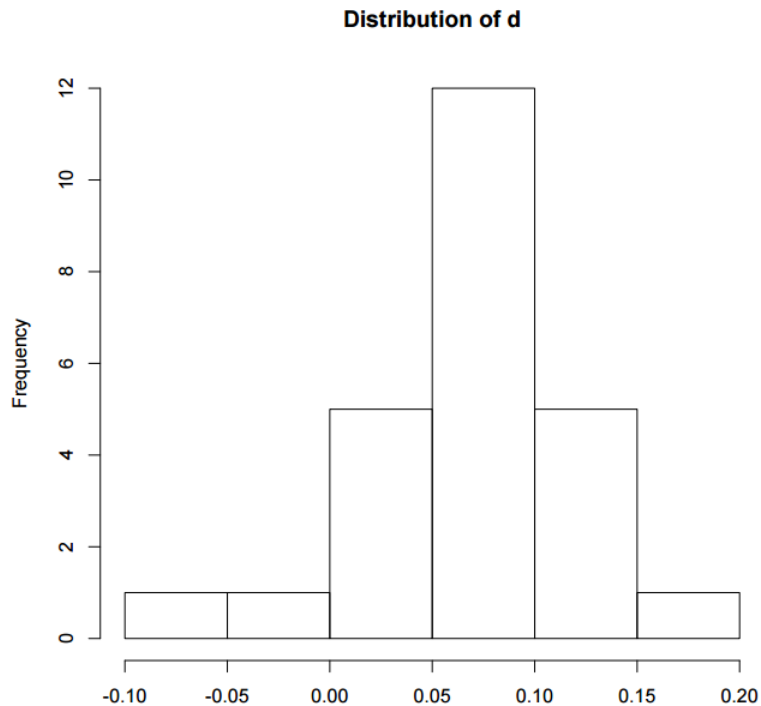


Figure 13: HISTOGRAM OF THE PARAMETER  $D$

In the distribution shown in Figure 13, it is possible to see that the mean of this parameter is not too much different from zero, which suggests a short memory process. The mean of this distribution is 0.07120278 and its standard deviation is 0.05292445.

Nonetheless, for comparison purposes another cellular automaton was also simulated to show that not all local interactions produce such long memory features. In this case, Rule 95 was simulated, with also 25 different initial conditions.

In this case, both GPH (Geweke; Porter-Hudak, 1983) and Künsch (1987) estimates were equal to one, suggesting a short memory process.

Hence, local interactions seem to be important characteristics of the system in order to exhibit long range dependency. But still, not all local interactions are able to produce such feature.

### **Sugarscape – A Simulation of an Artificial Society based on a Multi-Agent System**

The Sugarscape is a large scale agent-based model composed of (of course) agents, the environment (a two-dimensional grid) and a set of rules which governs the interactions between the agents and the environment. This model was originally presented in Epstein & Axtell (1996).

Each cell within the grid can contain different amounts of sugar and sugar capacity, where initially, the amount of sugar is equal to the sugar capacity. Whenever a patch is exploited, the amount of sugar is decreased, but it has a grow back constant, which allows restoring part of its sugar capacity.

These grid cells are randomly initialized in order to introduce spatial complexity in the simulation, as presented in Figure 14.

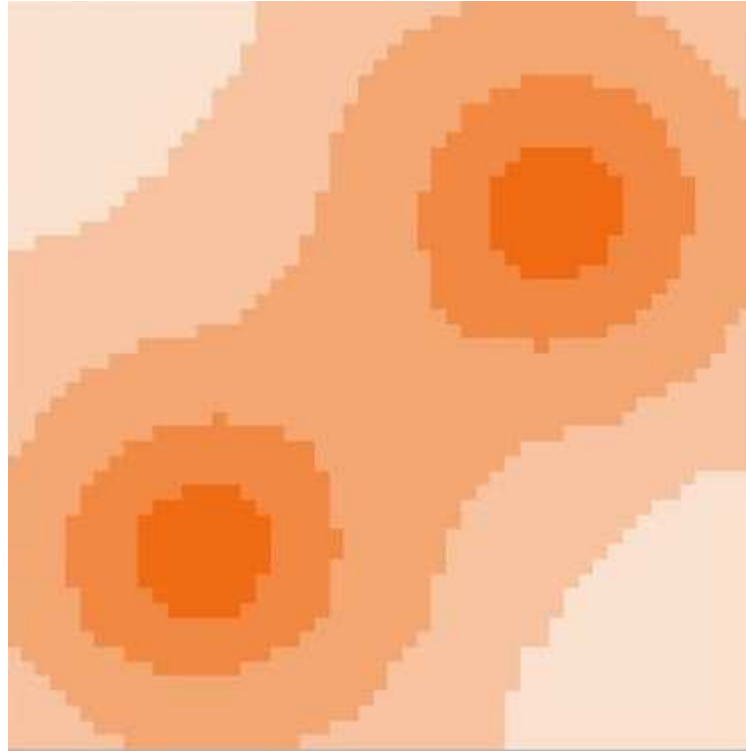


Figure 14: SUGAR SPATIAL DISTRIBUTION

Hence, as can be seen in Figure 14, the darker cells represent patches with higher sugar values, and the lighter ones patches with lower sugar values.

Moreover, this grid is populated by individual agents that have different states initialized randomly according to a uniform distribution:

- Amount of stocked sugar (defined by a range of Minimum and Maximum Initial Sugar Stocks)
- Metabolism (defined by a range of Minimum and Maximum Metabolism rates)
- Vision (defined by a range of Minimum and Maximum Vision capability)
- Life Expectancy (defined by a range of Minimum and Maximum Life Expectancy)

Hence, having these variables randomly initialized, the agent actions turn to be heterogeneous among themselves. Moreover, they are placed randomly within this grid, as is shown in Figure 15.

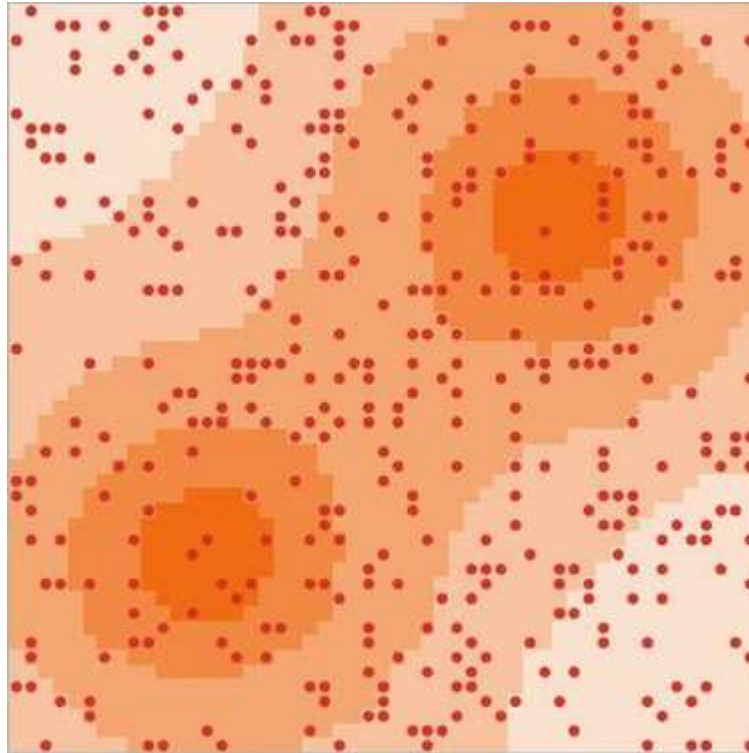


Figure 15: SPATIAL DISTRIBUTION OF ARTIFICIAL AGENTS

Given that, agents can search and gather sugar from cells within their vision range, they consume sugar from their stock according to their metabolism and they die if they run out of stock or if they achieve their life expectancy – when they die, they are replaced by other agents with random initial states.

Furthermore, they can explore only one cell grid at each tick and they select the cell grid according to the highest sugar value. If several patches exist with the same value, the agent chooses the closest one.

So, individual agents act with bounded rationality while exploiting the patches, given the fact that the choice of the patches is made according to the distance and the highest sugar value, but they do not coordinate their actions between themselves, which would be establishing who will explore a specific site, leading to suboptimal choices.

Finally, the execution order of this model is such that agents perform their respective operations according to a pre-specified set of rules, and then, all operations within each cell grid are carried out.

It is worth mentioning that there are also several other versions of this model that include more complex iterations between the agents and more spatial complexity, such as the introduction of another commodity (spice), combat rules between agents, sex and reproduction, genetic inheritance and so on,

but for the sake of this work, this basic set of rules exhibit the necessary features to study the role of spatial complexity in terms of rising long range dependency properties.

To achieve that, the present authors modified the original model in order to remove agent heterogeneity, by setting maximum and minimum life expectancy to the number of simulation ticks (avoiding the generation of new random agents by death cause), and the same for vision, initial quantity of sugar and metabolism. Hence, all agents behave the same way, and they do not get replaced by another agent (as in the original model) with a random set of characteristics which may introduce heterogeneity in the system.

Aiming comparisons between a complex environment and a simple environment, the present authors also modified the code to allow the removal of spatial complexity, by setting all patches identically and configuring the sugar restoration parameter to be larger than the agents' metabolism. Moreover, all heterogeneity between the patches is removed by imposing the same sugar capacity values for all of them – all patches having maximum capacity.

Consequently, in a first simulation, the system behaves like a stable intransient deterministic system.

After that, in a second simulation, the model is again modified, in order to generate spatial complexity. To achieve that, the sugar restoration and sugar capacity parameters are set to default values (identical to the original model), keeping all agents initially homogenous between themselves. Furthermore, heterogeneity is imposed over the patches by setting different and random sugar capacities, where only 5% of the patches have the maximum capacity.

Hence, this second configuration produces a result much similar to a stochastic process.

In order to check such results, a Gini coefficient time series was calculated over the food quantity that each agent has – in this case sugar is the Wealth in this simple artificial economy, simulated over 2000 periods.

Then, as can be seen in Figure 16, in the first discussed modification the system rapidly converges towards a fixed point. However, in the second configuration, the system behaves like a stochastic process, producing long memory properties that are going to be discussed later.

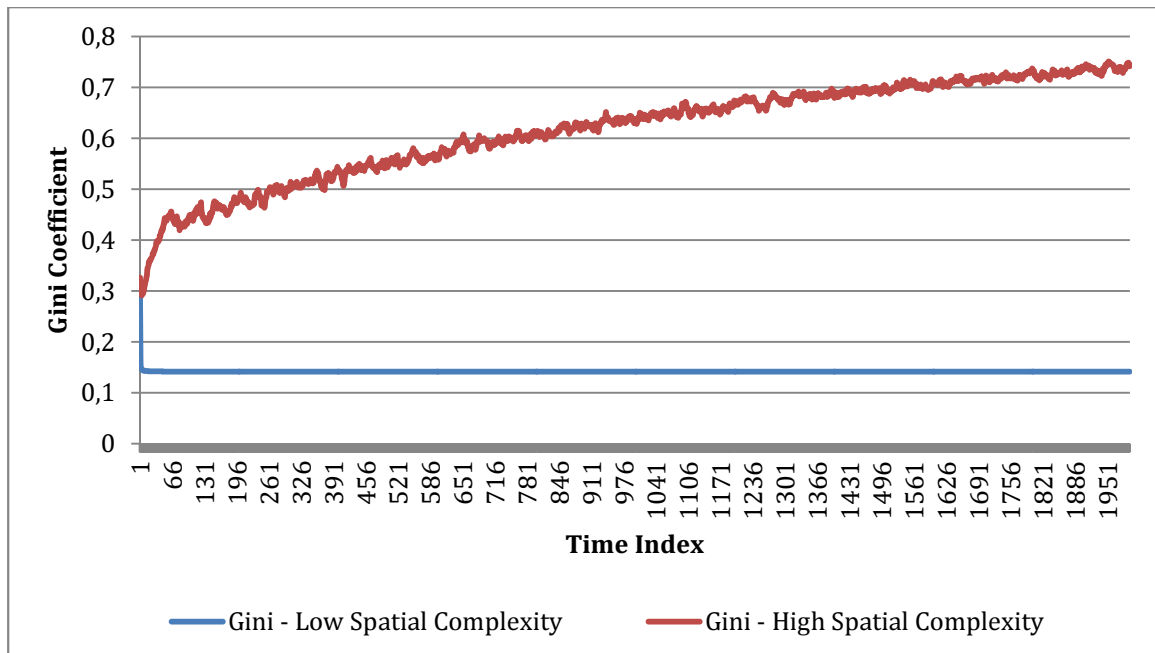


Figure 16: EVOLUTION OF SIMULATED GINI INDEXES

Keeping in mind such stochastic properties observed in the second configuration, the GPH (Geweke; Porter-Hudak, 1983) procedure was performed in order to analyze the long memory properties of such process and it was obtained the distribution presented in Figure 17, over 100 different experiments.

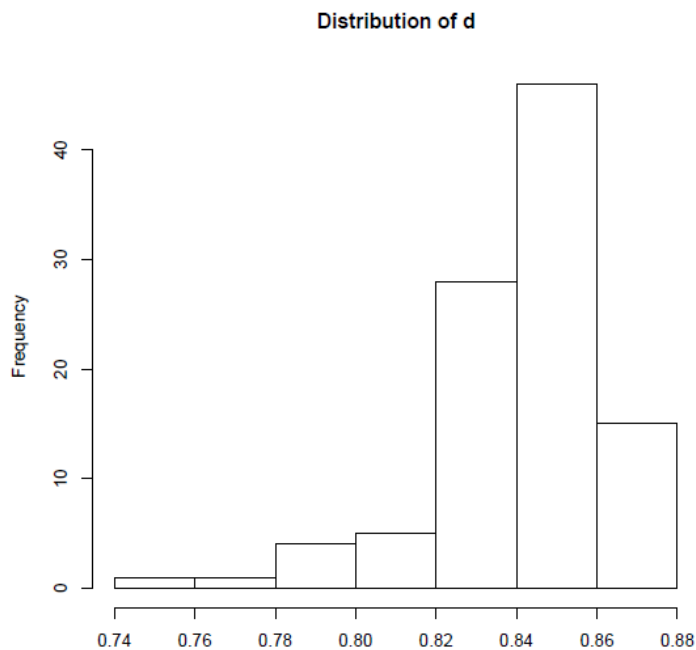


Figure 17: HISTOGRAM OF THE PARAMETER  $D$

As can be seen in Figure 17, there is a strong evidence of the presence of long range dependency in the analyzed stochastic process. In this specific case, the mean of this distribution is 0.8418075 and

its standard deviation is 0.02211333. It is also important to mention that the Künsch (1987) procedure failed in the R package and consequently did not produce any result that could be shown in this paper.

Therefore, from the case analyzed along this section, it is possible to show that spatial complexity plays a major role in the rise of long memory properties, when setting all agents behavior homogeneously and avoiding direct local interactions between themselves – in order to try to avoid all factors discussed previously.

### **Conclusions**

In this paper it was shown that it is possible to have long memory properties in variables that measures aggregate behavior of the agents, by analyzing the presence of the fractional difference parameter, as discussed in the traditional literature, without the need of economic/social frictions nor the presence of long memory behavior in the individual agents due to their respective biological processing unit capabilities (i.e. their brains).

Then, in order to show that, first, it was discarded the perspective of having spurious fractional difference coefficients derived from first order autoregressive structures, which may be present in computational representation of the individuals, due to the fact that the later can be thought as a linear approximation of the first case. Hence, it was demonstrated that only very high first order autoregressive behavior (parameters greater than 0.9) might lead to spurious long memory parameters. Keeping this in mind, the authors controlled the parameters of the first of three experiments to avoid any spurious long range dependency.

So, the authors conducted three different computational experiments from previous works, aiming to discuss the rise of long memory properties in the aggregate behavior of the agents, by simulating their individual actions and interactions. These three experiments were taken out in order to remove any possible perspective of conception biases, which may lead to the construction of systems with long memory properties. In other words, the present authors have chosen experiments built by other authors without such analysis perspective, in order to show that such properties may arise according to system features, instead of building models aiming the obtainment of long range dependency.

Thus, in the first experiment, adapted from Jumadinova, Matache & Dasgupta (2011), it was shown that long memory properties usually arise from heterogeneity between agents in a world where space and topology do not have any role.

In the second experiment, it was studied a class of individual computational simulations called Cellular Automata, which aims to complement the analysis done in the first experiment. Instead of having heterogeneous agents and a network that does not allow local interactions, it was chosen an experiment where all agents are homogenous (i.e. act the same way, using the same set of rules) and they interact locally, producing a global behavior. Consequently, it was shown that local interactions of individual homogeneous agents may produce long memory patterns observed in the global behavior.

Furthermore, in the third experiment, in order to complement both previous experiments, it was chosen another model – adapted from Epstein & Axtell (1996), where heterogeneity from the agents was removed and direct interactions between them were also removed. So basically, they only interact with a space that depends on the spatial configuration that the authors set up in order to control the presence of spatial complexity. Thus, such experiment suggests that spatial complexity is another interesting feature that plays an important role in the rise of long memory properties.

Therefore, keeping in mind these three different results, it is possible to show that long range dependency is a feature that emerges from the system complexity and not necessarily arises from individual long memory capabilities nor economic/social frictions and extends the idea of heterogeneity as a source of complexity, as usually discussed in the traditional economic literature (see Teyssière & Kirman, 2006).

As perspectives for future analysis and work, it is suggested expanding this analysis to other agent based models, setting up other possible sources of complexity within the system, in order to verify if such features appear or not.

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